

# CIVIL ENGINEERING

## Strength of Materials



Comprehensive Theory  
*with Solved Examples and Practice Questions*





**MADE EASY Publications Pvt. Ltd.**

**Corporate Office:** 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500

**Email :** infomep@madeeasy.in | **Web :** www.madeeasypublications.org

## Strength of Materials

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# CONTENTS

## Strength of Materials

### CHAPTER 1

#### Properties of Materials ..... 1-17

1.1	Introduction .....	1
1.2	Stress .....	1
1.3	Strain .....	3
1.4	Tensile Test for Mild Steel.....	4
1.5	Properties of Metals.....	7
1.6	Creep.....	7
1.7	Stress Relaxation.....	8
1.8	Elasticity.....	8
1.9	Toughness .....	10
1.10	Fatigue.....	10
1.11	Failure of Materials in Tension and Compression.....	11
	<i>Objective Brain Teasers</i> .....	13
	<i>Conventional Brain Teasers</i> .....	15

### CHAPTER 2

#### Simple Stress and Strain ..... 18-88

2.1	Stress .....	18
2.2	Strains .....	21
2.3	Matrix Representation of Stress and Strain.....	23
2.4	Differential Form of Strains .....	25
2.5	Allowable Stresses.....	26
2.6	Volumetric Strain ( $\bar{\epsilon}$ ) .....	27
2.7	Hooke's Law .....	27
2.8	Elastic Constants.....	28
2.9	Applications of Hooke's Law.....	30
2.10	Applications of Volumetric Strain .....	31
2.11	Deflection of Axially Loaded Members .....	34
2.12	Statically Indeterminate Axial Loaded Structures .....	47
2.13	Axial Deflection in Interconnected Members .....	51
2.14	Strain Energy .....	52
2.15	Thermal Stresses .....	56

2.16	Temperature Stresses in Composite Bar.....	62
2.17	Stresses in Bolts and Nuts.....	69
	<i>Objective Brain Teasers</i> .....	76
	<i>Conventional Brain Teasers</i> .....	85

### CHAPTER 3

#### Shear Force and Bending Moment ..... 89-177

3.1	Introduction .....	89
3.2	Supports .....	89
3.3	Beam .....	91
3.4	Loads.....	93
3.5	Stability in 2-D Structures.....	94
3.6	External Support Reactions in Beams .....	96
3.7	Shear Force and Bending Moment .....	97
3.8	Shear Force and Bending Moment Diagram .....	99
3.9	Curve Tracing for SFD and BMD .....	102
3.10	Example of Shear Force and Bending Moment Diagrams.....	103
3.11	Relationship between Load, Shear Force and Bending Moment .....	104
3.12	Important Points about Shear Force Diagrams and Bending Moment Diagrams Derived from Relationship.....	108
3.13	Maximum Bending Moment .....	108
3.14	SFD and BMD by Integration.....	114
3.15	Effect of Concentrated Moment on SFD and BMD... ..	125
3.16	Shear Force and Bending Moment Diagrams for Frames.....	131
3.17	Loading Diagram and BMD from SFD.....	137
3.18	Loading Diagram from BMD.....	142
3.19	Elastic Curves Using Bending Moment Diagram .....	144
	<i>Objective Brain Teasers</i> .....	158
	<i>Conventional Brain Teasers</i> .....	171

**CHAPTER 4****Bending Stress in Beams..... 178-234**

4.1	Introduction .....	178
4.2	Simple bending or pure bending .....	178
4.3	Nature of Bending Stress .....	184
4.4	Section Modulus (Z) .....	186
4.5	Moment of Resistance .....	190
4.6	Bending Stresses in Axially Loaded Beams .....	194
4.7	Force on a Partial Area of a Section.....	198
4.8	Composite Beams.....	200
4.9	Flitched Beam .....	202
4.10	Beam of Uniform Strength .....	211
4.11	Unsymmetrical Bending .....	212
4.12	Biaxial Bending.....	213
	<i>Objective Brain Teasers</i> .....	223
	<i>Conventional Brain Teasers</i> .....	231

**CHAPTER 5****Shear Stress in Beams ..... 235-265**

5.1	Introduction .....	235
5.2	Shear Stress in Beams .....	235
5.3	Analysis of shear stress in different sections .....	237
5.4	Shear Stresses in Composite Sections .....	247
5.5	Shear Centre .....	248
5.6	Shear Flow.....	251
5.7	Shear Centres of Thin-walled Open Sections .....	251
5.8	Shear Centres of Some Important Sections.....	255
	<i>Objective Brain Teasers</i> .....	259
	<i>Conventional Brain Teasers</i> .....	263

**CHAPTER 6****Transformation of Stresses ..... 266-329**

6.1	Introduction .....	266
6.2	Plane Stresses.....	266
6.3	Principal Stresses and Maximum Shear Stress.....	269
6.4	Principal Stresses in Beams .....	277
6.5	Mohr's Circle .....	280
6.6	Hooke's law for Plane Stress.....	286

6.7	Analysis of Strain.....	289
6.8	Transformation Equation for Plane Strain.....	290
6.9	Strain Energy .....	295
6.10	Strain Rosette.....	296
6.11	Theories of Elastic Failure .....	301
	<i>Objective Brain Teasers</i> .....	314
	<i>Conventional Brain Teasers</i> .....	323

**CHAPTER 7****Torsion of Shafts ..... 330-379**

7.1	Introduction .....	330
7.2	Difference between Bending Moment and Twisting Moment.....	330
7.3	Assumptions Involved in the Theory of Pure Torsion .....	330
7.4	Shear Stress Distribution in Circular Section .....	337
7.5	Design of Shaft .....	338
7.6	Power Transmitted by Shaft.....	339
7.7	Series Combination of Shaft.....	342
7.8	Parallel Combination of Shaft .....	342
7.9	Strain Energy in Torsion.....	345
7.10	Torsion in Thin Walled Tubes .....	347
7.11	Torsion of Non-circular Section .....	350
7.12	Indeterminate Shaft.....	350
7.13	Shaft Subjected to Combined Bending Moment and Twisting Moment.....	352
7.14	Shaft Subjected to Combined Axial Force and Torsional Moment .....	356
7.15	Theories of Failure for Shaft Design.....	358
	<i>Objective Brain Teasers</i> .....	368
	<i>Conventional Brain Teasers</i> .....	375

**CHAPTER 8****Deflection of Beams ..... 380-465**

8.1	Introduction .....	380
8.2	Double Integration Method .....	380
8.3	Moment Area Method (Mohr's Method).....	402
8.4	Conjugate Beam Method .....	419
8.5	Strain Energy Method .....	425

8.6	Method of Superposition .....	433
8.7	Application of Maxwell's Reciprocal Theorem .....	435
8.8	Slope and Deflection due to Temperature Change.....	437
	<i>Objective Brain Teasers</i> .....	448
	<i>Conventional Brain Teasers</i> .....	457

## CHAPTER 9

### Pressure Vessels ..... 466-500

9.1	Thin Cylindrical Shell .....	466
9.2	Analysis of Thin Cylindrical Shell with Closed Flat Ends .....	466
9.3	Strains in Cylindrical Shell .....	468
9.4	Analysis of Thin Spheres.....	472
9.5	Strains in Sphere .....	472
9.6	Stresses in Riveted Cylindrical Shell.....	473
9.7	Thin Cylinders with Hemispherical Ends.....	474
9.8	Pressure Vessels Subjected to Axial Force .....	475
9.9	Thick Cylinder .....	479
9.10	Analysis of Stresses .....	480
9.11	Analysis of Thick Sphere.....	482
9.12	Design of Pressure Vessels.....	484
9.13	Strengthening of Cylinder.....	485
	<i>Objective Brain Teasers</i> .....	490
	<i>Conventional Brain Teasers</i> .....	492

## CHAPTER 10

### Theory of Columns ..... 501-524

10.1	Compression Member .....	501
10.2	Types of Equilibrium.....	501
10.3	Euler's Theory for Buckling Failure.....	503
10.4	Maximum Lateral Deflection of Column.....	509
10.5	Rankine's Gordon Theory .....	510
10.6	Column with Eccentric Loading .....	512
10.7	Eccentric Loading about both x-axis and y-axis.....	515
	<i>Objective Brain Teasers</i> .....	518
	<i>Conventional Brain Teasers</i> .....	523

## CHAPTER 11

### Theory of Springs ..... 525-536

11.1	Springs .....	525
11.2	Types of Springs .....	525
11.3	Springs in Series and Parallel.....	529
	<i>Objective Brain Teasers</i> .....	532
	<i>Conventional Brain Teasers</i> .....	536





# Properties of Materials

## 1.1 INTRODUCTION

Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. A thorough understanding of mechanical behaviour is essential for the safe design of all structures, whether buildings, bridges, machines, motors, submarines or airplanes. Hence, strength of material is a basic subject in many engineering fields.

The objective of our analysis will be to determine the stresses, strains and deflections produced by the loads in different structures. Theoretical analysis and experimental results have equally important role in the study of strength of materials. So these quantities are found for all values of load upto the failure load, and then we will have a complete picture of the mechanical behaviour of the body.

The behaviour of a member subjected to forces depends not only on the fundamental law of Newtonian mechanics that govern the equilibrium of the forces but also on the mechanical characteristics of materials of which the member is fabricated. Sometimes, to predict the behaviour of material some necessary information regarding the characteristics of material comes from laboratory tests.

## 1.2 STRESS

The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial force  $P$  at the ends as shown.

A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the figure (a), axial force is acting away from the cross-section producing a uniform stretching of the bar, hence the bar is said to be in tension. Similarly in figure (c), axial force is acting towards the cross-section producing uniform compression of the bar, hence the bar is said to be in compression. To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section  $mn$  as shown in figure (b) and (d). This section is taken perpendicular to the longitudinal axis of bar. Hence it is known as cross-section.

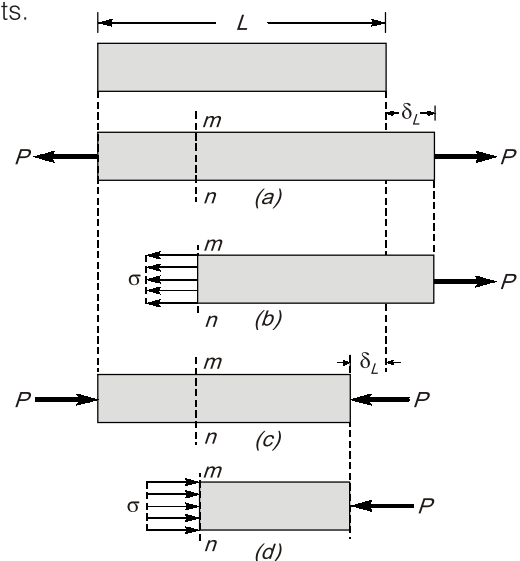


Fig. Axial stress

Now isolating the part of the bar to the right of the cut and considering the right of the cut as a free body. The force  $P$  has a tendency to move free body in the direction of load, so to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force developed, that is, internal force per unit area is called the **stress**.

Stress differs from pressure because pressure is defined as the externally applied force on unit area while stress is internal resistive force on unit area. To have better understanding of difference between externally applied force and internal resistance. Consider a bar suspended from a fixed end and a weight  $W$  is gradually applied at its free end as shown in figure.

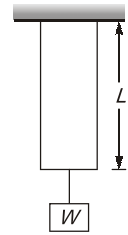


Fig. Axial load on bar

### Case-I: Weight, $W$ is applied gradually

Gradual loading means that value of load is zero at the starting time and gradually increases to value of  $W$ . Here, the bar gradually elongates with the increasing value of load. With increase in elongation, resistance forces say  $R$  will also increase gradually.

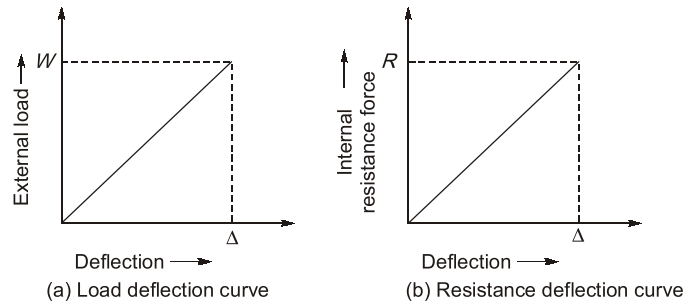


Fig. External load is applied gradually

### Case-II: Weight, $W$ is applied suddenly

Here, external load variation with elongation of bar is such as that its value instantly increases to  $W$ . This sudden load will result into elongation of bar say  $\Delta$ . When external load is applied suddenly, resistance force will be set up in bar, but unlike external load which is sudden, resistance force has always linear variation with elongation of bar.

Now, as clear from figure (a) and (b), intensity of pressure is not equal to stress induced in bar.

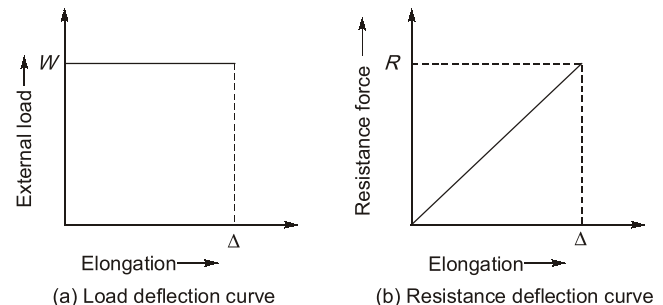


Fig. External load is applied suddenly

Thus, stress can be defined as – “**Stress is the internal resistance of a material offered against deformation which is expressed in terms of force per unit area**”.

Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **tensile** or **compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter  $\sigma$ . We can calculate stress mathematically as

$$\sigma = \frac{P}{A}$$

#### General Sign Convention:

Tensile stresses = +ve

Compressive stresses = -ve

**Unit:** (i)  $\text{N/m}^2$  or Pa (SI unit)

(ii)  $\text{N/mm}^2$  or MPa



- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed, then no stresses will be induced.
- Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.



On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

**(a) Engineering or nominal stress :** It is the stress where the original cross-sectional area of specimen is taken.

$$\text{Mathematically, } \sigma = \frac{P}{A_0}$$

where,  $A_0$  = Original cross-sectional area of specimen taken

**(b) True or actual stress :** It is the stress where the actual cross-sectional area of specimen at any time of loading is considered.

$$\text{Mathematically, } \sigma = \frac{P}{A_a}$$

where,  $A_a$  = Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading

$$A_a = A_0 \pm \Delta A \quad \text{as per our convention '+' for compression and '-' for tension is taken.}$$



**REMEMBER**

- In tension, true or actual stress is always greater than engineering or nominal stress.
- In compression, true or actual stress is always less than engineering or nominal stress.

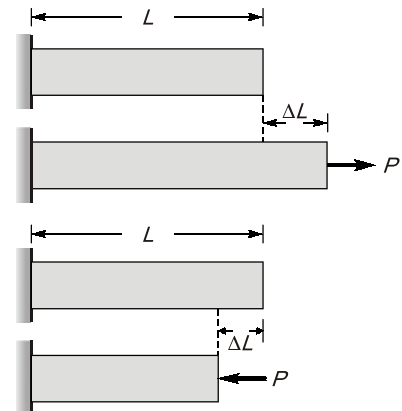
### 1.3 STRAIN

An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. The elongation or shortening in axially loaded member per unit length is known as strain. Strain is represented by  $\epsilon$ .

Mathematically, strain can be calculated as

$$\epsilon = \frac{\Delta L}{L}$$

Strain is dimensionless quantity and is always expressed in the form of number. If the member is in tension then the strain is called tensile strain. If the member is in compression, then the strain is called compressive strain.



*Fig. Strain in bars*

On the basis of length of member used in calculation of strain, strain can be of following two types:

**(a) Engineering or Nominal Strain:** Engineering or nominal strain is strain calculated, when length of member is taken as original length

$$\text{Mathematically, } \epsilon_0 = \frac{\Delta l}{l_0} \quad \text{where, } l_0 = \text{original length of member}$$

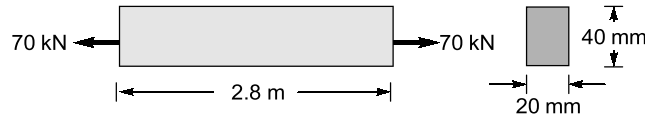
**(b) True or Actual Strain :** True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

$$\text{Mathematically, } \epsilon_a = \frac{\Delta l}{l_a} \quad \text{where, } l_a = \text{Actual length of member}$$

$$l_a = l_0 \pm \Delta l \quad \text{'+' sign for tension; '-' sign for compression}$$

**Example 1.1**

A prismatic bar with rectangular cross-section (20 mm × 40 mm), length  $L = 2.8$  m is subjected to an axial tensile force of 70 kN. The measured elongation of the bar is 1.2 mm. Calculate the tensile stress and strain in the bar.

**Solution:**

Assuming that force acts at CG of section. We know that,

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{70 \times 10^3 \text{ N}}{20 \times 40 \text{ mm}^2} = 87.5 \text{ N/mm}^2 = 87.5 \text{ MPa}$$

and

$$\text{Strain, } \epsilon = \frac{\Delta L}{L} = \frac{1.2 \text{ mm}}{2.8 \times 1000 \text{ mm}} = 4.286 \times 10^{-4}$$

## 1.4 TENSILE TEST FOR MILD STEEL

The mechanical properties of materials used in engineering are determined by experiments performed on small specimen. These experiments are conducted in laboratories equipped with testing machines that are capable of loading in tension or compression. The American Society for Testing and Materials (ASTM) has published guidelines for conducting test. Tensile test is generally conducted on Universal Testing Machine (UTM).

### 1.4.1 General Specifications of Specimen

- ♦ Specimen is solid cylindrical rod
- ♦ Gauge length 2" (inches)
- ♦ Diameter of middle section 0.5" (inches)
- ♦  $L/D$  ratio = 4.0

### 1.4.2 Stress Strain Curve for Tension

- ♦ **A is limit of proportionality:** Beyond this linear variation ceases. Hooke's law is valid in OA.
- ♦ **B is elastic limit:** The maximum stress upto which a specimen regains its original length on removal of applied load. For mild steel, B is very near to A. However, for other materials B may be greater than A.

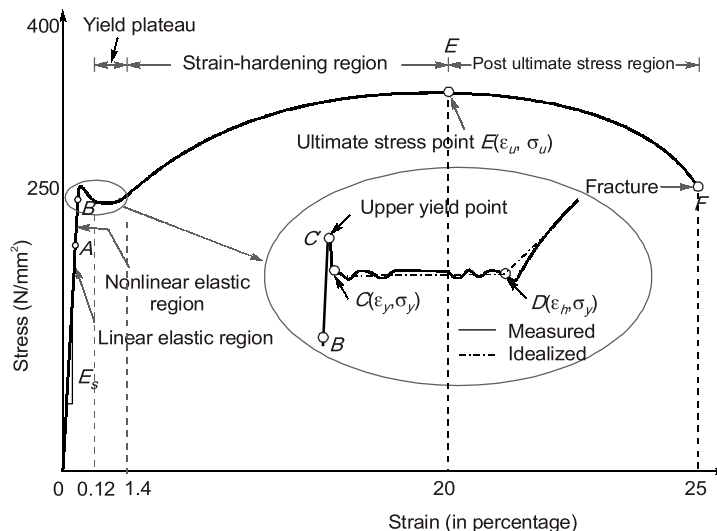


Fig. Ideal Tensile stress-strain diagram for Mild Steel

- ♦ **C' is upper yield point:** The magnitude of the stress corresponding to C' depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.
- ♦ **C is lower yield point:** This is also called actual yield point. The stress at C is the yield stress ( $\sigma_y$ ) with a typical value of  $\sigma_y = 250 \text{ N/mm}^2$  (for mild steel). The yielding begins at this stress.
- ♦ **CD represents perfectly plastic region:** It is the strain which occurs after the yielding point C, without any increase in stress. The strain corresponding to point D is about 1.4% and corresponding to C is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- ♦ **DE represents strain hardening region:** In this range further addition of stress gives additional strain. However, strain increases with faster rate in this region. The material in this range undergoes change in its crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- ♦ **E is ultimate point:** The stress corresponding to this point is ultimate stress ( $\sigma_u$ ) and the corresponding strain is about 20% for mild steel.
- ♦ **F is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- ♦ **EF post ultimate stress region:** In this range, necking occurs, i.e. area of cross-section is drastically decreased.

**NOTE**



1. Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.
2. Ideal curve for tension is shown in figure. However actual behaviour is different and indicates apparently reduced yield stress in compression after strain hardening in tension. The divergence between tension and compression results is explained by Bauschinger and is called **Bauschinger effect**.

### 1.4.3 Actual Curve v/s Engineering Curve in Tension and Compression for Mild Steel

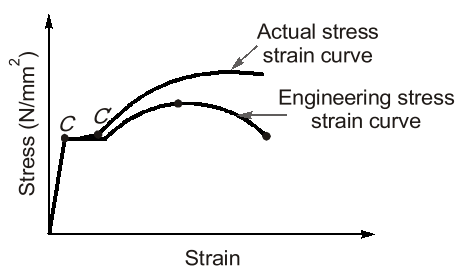


Fig. Tension curve for mild steel

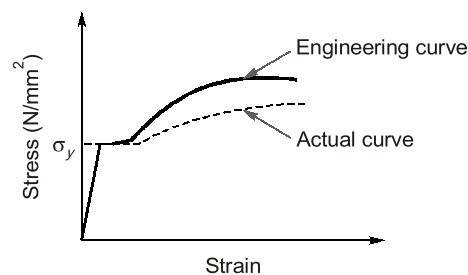


Fig. Compression curve for Mild steel

**NOTE**



- The fracture strain depends upon % carbon present in steel.
- With increase in percentage carbon, fracture strain reduces.
- With increase in carbon content, steel has higher yield stress and higher ultimate stresses.

- ♦ In compression, engineering stress-strain curve lies above the actual stress-strain curve, while in tension actual stress-strain curve lies above the engineering stress-strain curve.
- ♦ In compression mild steel has yield stress  $\sigma_y = 263 \text{ N/mm}^2$ , slightly greater than tension.
- ♦ Mild steel has same Young's modulus of elasticity in compression and tension,  $E = 2.1 \times 10^5 \text{ N/mm}^2$ .

### Relation between engineering and actual stress

$$\sigma_a = \sigma_0(1 \pm \epsilon_0)$$

where,  $\sigma_a$  = Actual stress;  $\sigma_0$  = Engineering stress;  $\epsilon_0$  = Engineering strain

As per our convention, for tension, take positive (+ve) sign and take negative (-ve) sign for compression.'

**NOTE :** While deriving above equation, volume changes is neglected which is true in plastic region (Non-elastic region).

### 1.4.4 Stress-strain Curve for other Grades of Steel in Tension

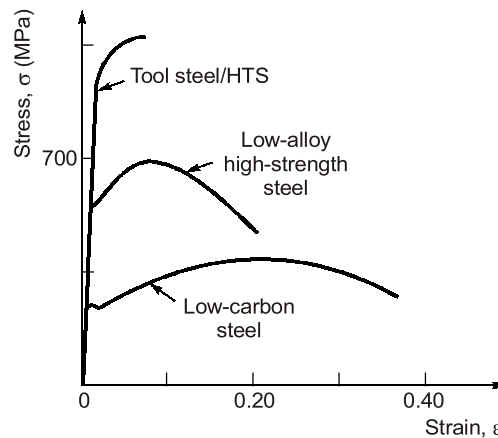


Fig. Tensile stress-strain diagram for different grades of steel



#### REMEMBER

- All the grades of steel have same Young's modulus of elasticity.
- Among all steel grades high tension steel (HTS) is more brittle and mild steel is more ductile.
- High tension steel has higher ultimate strength than other grades of steel.

### 1.4.5 Stress-strain Curve for Different Materials

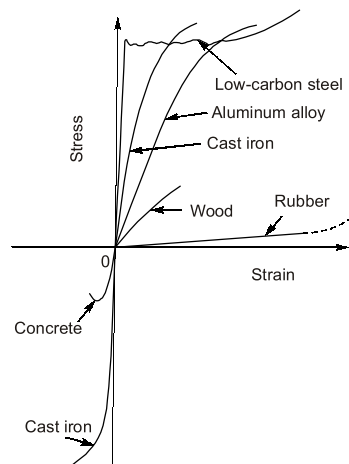


Fig. Stress-strain diagram for different material

## 1.5 PROPERTIES OF METALS

### 1.5.1 Ductility

Ductility is the property by which material can be stretched. Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place. These materials have post-elastic strain (Plastic strain) greater than 5%. Some of the examples of ductile materials are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze etc.

### 1.5.2 Brittleness

Brittleness is the lack of ductility i.e. materials can not be stretched. In brittle materials, fracture takes place immediately after elastic limit with a relatively smaller deformation. For the brittle materials, fracture and ultimate points are same and after proportional limit very small strain is seen. Brittle materials have post elastic strain less than 5%. Some examples of brittle materials are cast iron, concrete and glass.



**REMEMBER**

To distinguish between these two type of materials, materials with post elastic strain less than 5% at fracture point are regarded as brittle and those having post elastic strain greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

### 1.5.3 Malleability

Malleability is the property of metal due to which a piece of metal can be converted into a thin sheet by pressing it. A malleable material possess a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

### 1.5.4 Hardness

- ♦ Hardness is resistance to scratch or abrasion.
- ♦ There are two methods of hardness measurements:
  - (a) Scratch hardness-commonly measured by Mohr's test
  - (b) Indentation hardness (abrasion) measured by
    - Brinell's hardness method
    - Vickers hardness
    - Rockwell hardness
    - Knoop hardness

## 1.6 CREEP

Creep is permanent deformation which is recorded with passage of time at constant loading. Total creep deformation continues to increase with time asymptotically. Consider a prismatic bar of length L on which an external static load P is applied. Due to applied static load, goes a deformation of  $\Delta_e$ , but after some time it is observed that bar has gone permanent deformation and some stress developed in bar released. This effect is called creep.

where,

$$\Delta_e = \text{Elastic deflection} = \frac{PL}{AE}$$

$P$  = Static load

$\Delta_c$  = Deformation due to creep

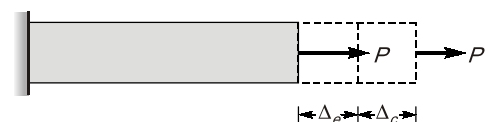


Fig. Creep in bar

Factors affecting creep are as follows:

1. Magnitude of load
2. Type of loading (static or dynamic)
3. Time or age of loading
4. Temperature
  - ♦ At higher temperature, due to greater mobility of atoms most of the materials lose their strength and elastic constants also get reduced. Hence, greater deformation at elevated temperature results, even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
  - ♦ Temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and is known as **homologous temperature**.

## 1.7 STRESS RELAXATION

If a wire of metal is stretched between two immovable supports, so that it has an initial tension  $\sigma_0$ . The stress in the wire gradually diminishes, eventually reaching a constant value. This process, which is manifestation of creep is called **stress relaxation**. (This is the reason why electric wires sag after long time)

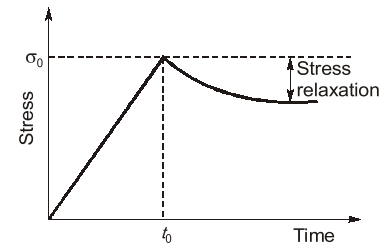


Fig. Stress relaxation

## 1.8 ELASTICITY

Assume for instance, we apply a tensile load to a specimen so that strain and stress follow path from  $O$  to  $B$  on stress-strain curve shown in figure. Further, when the load is removed, the material follows exactly the same curve back to the origin  $O$ . So, the property by which original dimensions (i.e. length and cross-section) can be recovered after unloading is known as **elasticity**.

Within elastic limit curve may be linear or non-linear. During loading, material store elastic strain energy. The total strain energy which can be stored in the given volume of the metal and can be released after unloading is called **resilience**. It is also equal to area under load deflection curve within elastic limit ( $B$ ). When elastic limit coincides with yield point, the maximum elastic energy per unit volume is known as **modulus of resilience**.

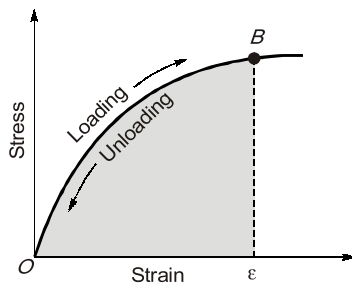


Fig. Stress strain curve

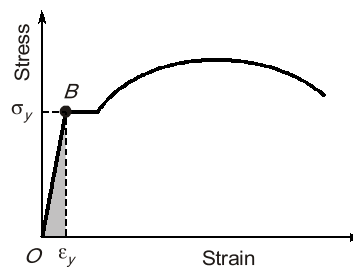


Fig. Stress strain curve in mild steel

Modulus of resilience is equal to area under the stress-strain curve within elastic limit for mild steel.

$$U_r = \frac{1}{2} \times \sigma_y \times \epsilon_y$$

**NOTE**



- The modulus of resilience depends upon yield strength and hence a material with higher yield strength will have higher modulus of resilience.
- Higher resilience is desirable in suspension spring and where the load absorption is required.
- HTS (High Tension Steel) has more yield stress than mild steel so it has more modulus of resilience. Thus springs are made from high tension steel.

**1.8.1 Proof Stress**

Some of the ductile metals like Aluminium (Al), Copper (Cu) and Silver (Ag) do not show clear yield point in tension test, therefore, their yield stress ( $\sigma_y$ ) is not clearly known. For such metals, design stress is calculated by offset method. An offset of permanent plastic strain equals to 0.2% generally is marked on  $x$ -axis and a straight line is drawn which is parallel to initial portion of stress-strain curve. The point of intersection of stress-strain curve with straight line is called proof point and the corresponding stress at that point is **called proof stress**.

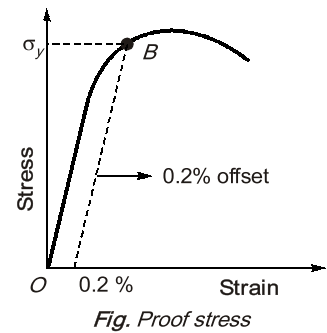


Fig. Proof stress

**1.8.2 Elasto-Plastic Behaviour of Metals**

Now, suppose that material is loaded to a much higher level than elastic limit ( $B$ ), such that point  $P$  is reached on stress-strain diagram. When unloading occurs the material follows path  $PC$  as shown in the figure, which is parallel to the initial portion of original stress-strain curve. When point  $C$  is reached, the load has been entirely removed but a permanent strain or residual strain  $OC$  remains in material. The corresponding residual elongation of the specimen is called **permanent set**.

During unloading, only  $CPD$  part of strain energy is recovered and is called as elastic strain energy, whereas a large part  $OPC$  is lost in permanent deformation and is called **inelastic strain energy**.

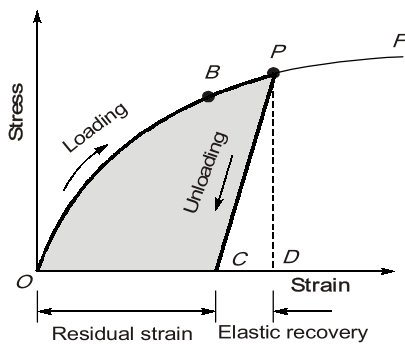


Fig. Partially elastic behaviour

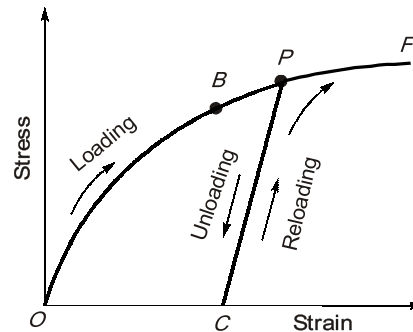
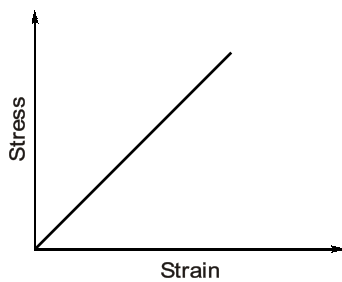


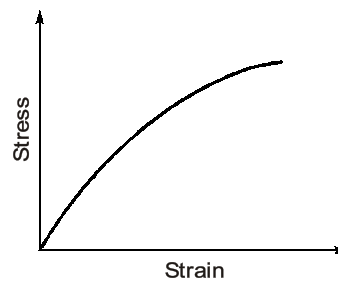
Fig. Reloading of a material and raising of the yield stress

If material undergoes continuous cyclic loading and unloading beyond elastic limit, then yield limit of material continuously increases. This concept is used in cold working of mild steel bar to avoid yield plateau.

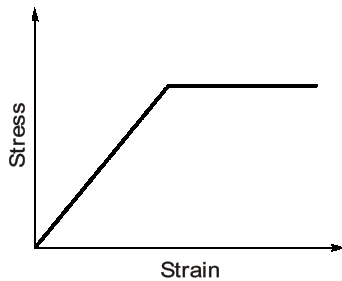
**1.8.3 Types of Material Behaviour**



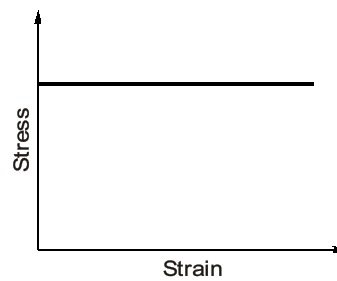
(i) Linear elastic



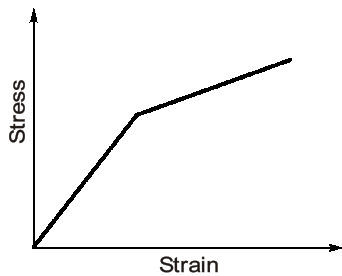
(ii) Non-linear elastic



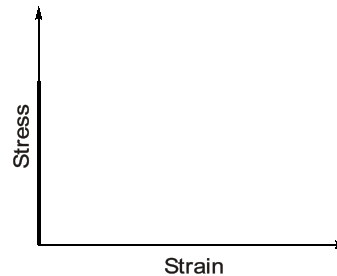
(iii) Elasto plastic or visco-plastic



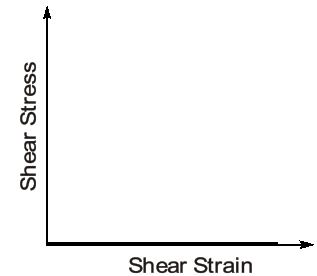
(iv) Perfectly plastic



(v) Elasto-plastic with strain hardening



(vi) Ideal rigid



(vii) Ideal fluid

## 1.9 TOUGHNESS

The property which enables material to absorb energy without fracture. This property is very desirable in case of cyclic loading or shock loading. If a material is tough, then it has the ability to store large strain energy before fracture. **Modulus of toughness** is total strain energy per unit volume upto fracture stage. It is equal to total area under stress-strain curve upto fracture.

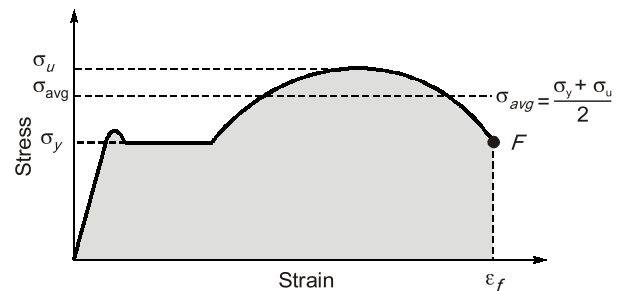


Fig. Area under stress strain upto Fracture

$$\text{Modulus of toughness} = \left( \frac{\sigma_y + \sigma_u}{2} \right) \times \epsilon_f$$

where,  $\sigma_y$  = yield tensile strength,  $\sigma_u$  = ultimate tensile strength and  $\epsilon_f$  = strain at fracture point fracture.

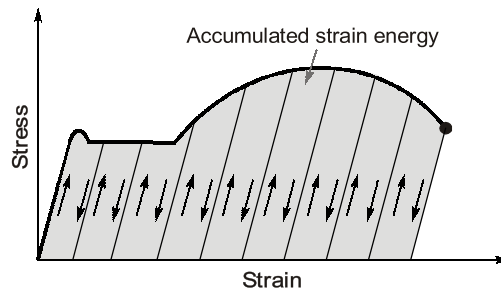
The modulus of toughness depends upon ultimate tensile strength and strain at failure (Fracture strain). Hence, the material which is very ductile will exhibit a higher modulus of toughness as is the case with mild steel.

**Remember:** Ductile materials are tough and brittle materials are hard.

## 1.10 FATIGUE

It has been found that material behave differently under the static and dynamic loading. In cyclic or reverse cyclic loading, if total accumulated strain energy exceed the toughness, then fracture failure may occur.





Factors affecting fatigue are:

1. Loading condition
2. Frequency of loading
3. Corrosion
4. Temperature
5. Stress concentration

The number of load cycles required to initiate surface crack is called **fatigue initiation life** and additional number of load cycle required to propagate surface crack is called **fatigue propagation life**.

To prevent fatigue failure the developed stress should be kept below endurance limit. **Endurance limit** is that stress below which a material has high probability of no failure even at infinite number of load cycles.

For, mild steel, endurance limit is  $186 \text{ N/mm}^2$  and for Aluminium, endurance limit is  $131 \text{ N/mm}^2$

**Remember:** Endurance limit is lower than the proportional limit.

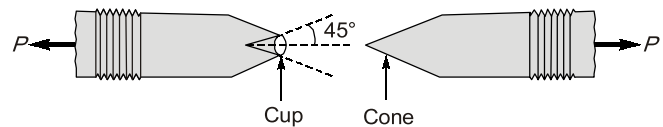
Some examples of fatigue failure are:

1. Crashing of aircraft due to crack in turbine blade
2. Failure of fly wheels
3. Breaking of wire due to cyclic bending

## 1.11 FAILURE OF MATERIALS IN TENSION AND COMPRESSION

### 1.11.1 Ductile Materials in Tension Test

Ductile materials are weak in shear and failure is due to shear strain along the plane forming  $45^\circ$  angles with the axis of the specimen. In ductile material, cup and cone fracture take place. Failure surface is rough.

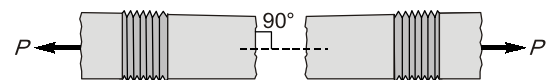


*Fig. Fracture in ductile materials*

**NOTE :** In ductile materials, necking is form before fracture.

### 1.11.2 Brittle Materials in Tension Test

Brittle materials are very weak in tension. Brittle materials fail due to separation of particles along the surface which is at  $90^\circ$  to the direction of load. Failure surface is rough.



*Fig. Brittle failure of material*

### 1.11.3 Ductile Materials in Compression Test

Short compression members fail in compression yielding. Failure plane is parallel to the compressive load. In compression yielding, bulging of material occurs as shown in figure which leads to crack formation in a direction to compressive load.

### 1.11.4 Brittle Materials in Compression Test

In compression, brittle materials fail in shear, failure plane is at  $45^\circ$  to the direction of loading as shown in figure.

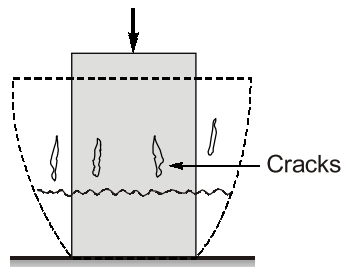


Fig. Ductile failure in compression

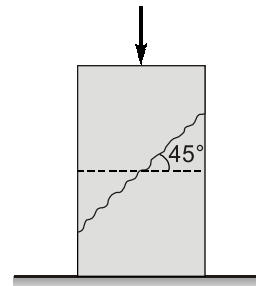
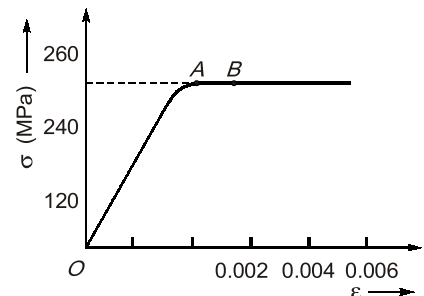


Fig. Brittle failure in compression

#### Example 1.2

A bar of length 5.0 m is made of a structural steel having the stress-strain diagram shown in the Figure. The yield stress of the steel is 250 MPa and the slope of the initial linear part of the stress-strain curve (Modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 7.5 mm and then the load is removed. How does the final length of the bar compare with its original length?



#### Solution:

Given: Length of specimen,  $L = 5000$  mm  
Yield stress,  $\sigma_y = 250$  MPa  
Modulus of elasticity,  $E = 200$  GPa  
Elongation,  $\Delta L = 7.5$  mm

Total strain at  $B$  due to axial loading,

$$\epsilon_B = \frac{\Delta L}{L} = \frac{7.5 \text{ mm}}{5000 \text{ mm}} = 0.0015$$

Elastic strain at  $A$ ,

$$\epsilon_A = \frac{\sigma_y}{E} = \frac{250 \text{ MPa}}{2 \times 10^5 \text{ MPa}} = 0.00125$$

It is clear that  $\epsilon_B > \epsilon_A$ , it means specimen is loaded beyond elastic limit and on unloading total dimension will not recover and permanent elongations will occur.

So, elastic recovery due to unloading

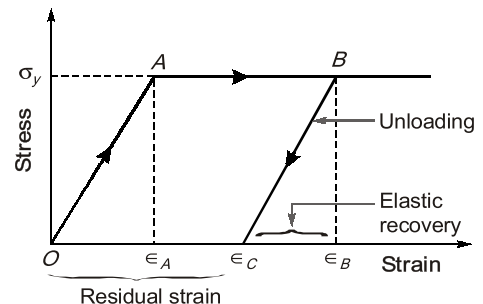
$$\text{Elastic recovery} = \frac{\sigma_y}{E} = 0.00125$$

so permanent strain or residual strain

$$\begin{aligned} \epsilon_C &= \epsilon_B - \epsilon_{\text{Recovery}} = 0.0015 - 0.00125 \\ &= 0.00025 \end{aligned}$$

so permanent deformation,

$$\Delta L_p = \epsilon_C \times L = 0.00025 \times 5000 = 1.25 \text{ mm (longer)}$$



◆◆◆◆



**OBJECTIVE  
BRAIN TEASERS**

- Q.1** Match List-I (Type of material) with List-II (Characteristics) and select the correct answer using the codes given below the lists:
- List-I**
- Elastic material
  - Rigid material
  - Plastic material
  - Resilient material
- List-II**
- Does not store energy
  - Has no plastic region in stress strain curve
  - Behave as a spring
  - Offers resistance to deformation
  - Does not offer resistance to deformation
- Codes:**
- |     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 4 | 1 | 5 | 3 |
| (b) | 4 | 1 | 5 | 2 |
| (c) | 2 | 3 | 4 | 5 |
| (d) | 1 | 3 | 4 | 5 |
- Q.2** A structure is said to be linearly elastic if
- Load  $\propto$  displacement
  - Load  $\propto \frac{1}{\text{Displacement}}$
  - Energy  $\propto$  displacement
  - Energy  $\propto$  Load
- Q.3** Which one is not the characteristics of fatigue fracture?
- Rough fracture surface
  - Rough and smooth areas on fracture surface
  - Plastic deformation
  - Conchoidal markings on fracture surface
- Q.4** Stress curve is always a straight line for
- elastic material
  - materials obeying Hooke's law
  - elasto plastic materials
  - none of the above
- Q.5** Consider the following statements:  
The principle of superposition is applied to
- Linear elastic bodies
  - Bodies subjected to small deformations
- Which of these statements is/are correct?
- 1 alone
  - 1 and 2
  - 2 alone
  - neither 1 nor 2
- Q.6** The strain at a point is a
- Scalar
  - Vector
  - Tensor
  - None of these
- Q.7** Consider the following statements
- Strength of steel increases with carbon content
  - Young's modulus of steel increase with carbon content
  - Young's modulus of steel remain unchanged with variation of carbon content
- Which of these statements is/are correct?
- 1 only
  - 2 only
  - 1 and 2
  - 1 and 3
- Q.8** True stress  $\sigma$  is related with conventional stress  $\sigma_0$  as
- $\frac{\sigma}{\sigma_0} = (1 + \epsilon)^2$
  - $\frac{\sigma}{\sigma_0} = \frac{1}{(1 + \epsilon)^2}$
  - $\frac{\sigma}{\sigma_0} = \frac{1}{(1 + \epsilon)}$
  - $\frac{\sigma}{\sigma_0} = 1 + \epsilon$
- Q.9** Steel has its yield strength of 400 N/mm<sup>2</sup> and modulus of elasticity of  $2 \times 10^5$  MPa. Assuming the material to obey Hooke's law up to yielding, what is its proof resilience?
- 0.8 N/mm<sup>2</sup>
  - 0.4 N/mm<sup>2</sup>
  - 0.6 N/mm<sup>2</sup>
  - 0.7 N/mm<sup>2</sup>
- Q.10** What would be the shape of the failure surface of a standard cast iron specimen subjected to torque?
- Cup and cone shape at the center.
  - Plane surface perpendicular to the axis of the specimen.
  - Pyramid type wedge-shaped surface perpendicular to the axis of the specimen.
  - Helicoidal surface at 45° to the axis of the specimen.

- Q.11** The greatest stress that a material can withstand for a specified length of time without excessive deformation is called
- Fatigue strength
  - Endurance limit
  - Creep rupture strength
  - Creep strength

- Q.12** Consider the following statements:
- Creep is usually more important at higher temperatures and higher stress.
  - Creep depends on temperature level, stress level, time and type of loading (static or dynamic).
  - The nature of creep is elastic because as soon as we remove the load, the material regains its original length.
  - Creep in concrete may relieve some tensile stress.

Which of these statements are correct?

- 1, 2 and 3
- 1, 3 and 4
- 1, 2 and 4
- 2 and 4 only

- Q.13** The following observations refer to two metal samples *A* and *B* of same size subjected to uni-axial tension test upto failure
- Elastic strain energy of *A* is more than that of *B*.
  - Area under stress-strain curve of *A* is less than that of *B*.
  - The yield strength of *A* is more than that of *B*.
  - The percentage elongation of *A* and *B* at elastic limit are equal.

Which of the following statement is true in this regard?

- Specimen *A* is more ductile than specimen *B*
- Specimen *B* is more ductile than specimen *A*
- The ductility of the two specimen is equal
- The data is insufficient to compare the ductilities of the two specimens

- Q.14** What is the number of independent stress components in a body loaded under a general state of stress and a plane stress condition respectively in order to completely specify the state of stress at a point?

- 9 and 4
- 6 and 4
- 9 and 3
- 6 and 3

**Directions:** The following items consists of two statements; one labelled as '**Assertion (A)**' and the other as '**Reason (R)**'. You are to examine these two statements carefully and select the answers to these items using the codes given below:

- both **A** and **R** are true and **R** is the correct explanation of **A**
- both **A** and **R** are true but **R** is not a correct explanation of **A**
- A** is true but **R** is false
- A** is false but **R** is true

**Q.15 Assertion (A):** Many materials do not have well defined yield point.

**Reason (R):** 0.2% offset parallel to the initial tangent of the stress-strain curve intersects the curve at yield stress.

**Q.16 Assertion (A):** The amount of elastic deformation at a certain point, which an elastic body undergoes, under given stress is the same irrespective of the stresses being tensile or compressive.

**Reason (R):** The modulus of elasticity and Poisson's ratio are assumed to be the same in tension as well as compression.

**Q.17 Assertion (A):** The failure surface of a mild steel specimen subject to a torque about its axis is along a surface perpendicular to its axis.

**Reason (R):** Mild steel is relatively weaker in shear than in tension and the plane of maximum shear is perpendicular to its axis.

**Q.18 Assertion (A):** In strain hardening region, the material appears to lose some of its strength and hence offers more resistance, thus requiring increased tensile load for further deformation.

**Reason (R):** The material undergoes changes in its atomic and crystalline structure in this region.

**Q.19 Assertion (A):** There are two independent elastic constants for an isotropic material.

**Reason (R):** All metals at micro-level are isotropic.

■■■■